nag_deviates_chi_sq (g01fcc)

1. Purpose

nag_deviates_chi_sq (g01fcc) returns the deviate associated with the given lower tail probability of the χ^2 distribution with real degrees of freedom.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_deviates_chi_sq(double p, double df, NagError *fail)
```

3. Description

The deviate, x_p, associated with the lower tail probability p of the χ^2 distribution with ν degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{v/2-1}\, dX \qquad 0 \leq x_p < \infty; \quad \nu > 0.$$

The required x_p is found by using the relationship between a χ^2 distribution and a gamma distribution, i.e., a χ^2 distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

For very large values of ν, greater than 10^5, Wilson and Hilferty's normal approximation to the χ^2 is used, see Kendall and Stuart (1969).

4. Parameters

p
Input: the probability, p, from the required χ^2 distribution.
Constraint: $0.0 \leq \mathbf{p} < 1.0$.

df
Input: the degrees of freedom, ν, of the χ^2 distribution.
Constraint: **df** > 0.0.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

On any of the error conditions listed below except **NE_ALG_NOT_CONV** nag_deviates_chi_sq returns 0.0.

NE_REAL_ARG_LT
On entry, **p** must not be less than 0.0: **p** = ⟨*value*⟩.

NE_REAL_ARG_GE
On entry, **p** must not be greater than or equal to 1.0: **p** = ⟨*value*⟩.

NE_REAL_ARG_LE
On entry, **df** must not be less than or equal to 0.0: **df** = ⟨*value*⟩.

NE_PROBAB_CLOSE_TO_TAIL
The probability is too close to 0.0 or 1.0.

NE_ALG_NOT_CONV
The algorithm has failed to converge in ⟨*value*⟩ iterations.
The result should be a reasonable approximation.

NE_GAM_NOT_CONV
The series used to calculate the gamma probabilities has failed to converge.
This is an unlikely error exit.

6. Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to nag_deviates_gamma_dist (g01ffc) made.

6.1. Accuracy

The results should be accurate to 5 significant digits for most parameter values. Some accuracy is lost for p close to 0.0.

6.2. References

Best D J and Roberts D E (1975) The percentage points of the χ^2 distribution *Appl. Stat.* **24** Algorithm AS91 385–388.
Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.
Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Vol 1)* Griffin.

7. See Also

nag_deviates_gamma_dist (g01ffc)

8. Example

Lower tail probabilities are read for several χ^2 distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

8.1. Program Text

```
/* nag_deviates_chi_sq(g01fcc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
  double df, p, x;
  static NagError fail;

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("g01fcc Example Program Results\n");
  Vprintf("     p       df      x\n\n");
  while (scanf("%lf %lf", &p, &df) != EOF)
    {
      x = g01fcc(p, df, &fail);
      if (fail.code==NE_NOERROR)
        Vprintf("%8.3f%8.3f%8.3f\n", p, df, x);
      else
        Vprintf("%8.3f%8.3f%8.3f\n Note: %s\n", p, df, x, fail.message);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01fcc Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0
```

8.3. Program Results

```
g01fcc Example Program Results
     p       df      x

   0.010  20.000   8.260
   0.428   7.500   6.200
   0.869  45.000  55.759
```